Bosonic fields in the stringlike defect model

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We study localization of bosonic bulk fields on a stringlike defect with codimension 2 in a general space-time dimension in detail. We show that in cases of spin-0 scalar and spin-1 vector fields there are an infinite number of massless Kaluza-Klein states which are degenerate with respect to the radial quantum number, but only the massless zero-mode state among them is coupled to the fermion on the stringlike defect. We also comment on interesting extensions of the model at hand to various directions such as ''little'' superstring theory, conformal field theory, and a supersymmetric construction.

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I. INTRODUCTION

In theories where our four dimensional world is a three-brane embedded in a higher-dimensional space [1–5], the conventional Kaluza-Klein scenario would be modified drastically. In most works on Kaluza-Klein compactification thus far, a higher-dimensional manifold is assumed to be composed of as a direct product of a noncompact four-dimensional Minkowski space-time and a compact internal manifold with the size of the compact space being set by the Planck scale. However, in this approach it seems to be quite difficult to stabilize the size of all the internal dimensions around the Planck scale via some nonperturbative effects. This problem should be solved in the brane world.[1]

In recent years, an alternative scenario of compactification has been put forward [5]. This new idea is based on the possibility that our world is a three-brane embedded in a higher-dimensional space-time with a nonfactorizable warped geometry. In this scenario, we are free from the moduli stabilization problem in the sense that the internal manifold is noncompact and does not need to be compactified to the Planck scale anymore, which is one of reasons why this new compactification scenario has attracted so much attention. An important ingredient of this scenario is that all the matter fields are thought of as confined to the three-brane, whereas gravity is free to propagate in the extra dimensions. Such localization of matter would be indeed possible in D-brane theory [6] and M theory [7], but at present it is far from complete to realize the Rundall-Sundrum model [5] within the framework of superstring theory. Thus, it is worthwhile to explore whether such localization is also possible in local field theory.

In fact, the localization mechanism has been recently investigated in AdS_5 space [8–12]. In particular, it is shown that the spin-0 field is localized on a brane with positive tension which also localizes the graviton [11], while the spin-1 field is not localized either on a brane with positive tension or on a brane with negative tension [9,11]. Moreover, it is shown that spin-1/2 and -3/2 fields are localized not on a brane with positive tension but on a brane with negative tension [10,11]. Thus, in order to satisfy the localization of standard model particles on a brane with positive tension, it seems that some additional interactions except gravity must be also introduced in the bulk.

More recently, the possibility of extending the Randall-Sundrum domain wall model to higher-dimensional topological objects was explored [13–23]. In particular, we find that Einstein's equations admit a stringlike defect with codimension 2 in addition to a domain wall with codimension 1.[2] In particular, the existence of the stringlike defect makes it possible to think of a three-brane in six-dimensional anti–de Sitter space.

In this stringlike defect model, the localization of bulk fields has been also investigated. In Ref. [18], it is shown that the spin-2 graviton is localized on the three-brane and the corrections to Newton's law are more supressed than in the domain wall model. Afterwards, the present author explored the localization of various spin fields on the stringlike defect in a general dimension and obtained the following facts [22]: spin-0, -1, and -2 bosonic fields are localized on a stringlike defect with the exponentially decreasing warp factor, whereas spin-1/2 and3/2 fermionic fields are localized on a defect with the exponentially increasing warp factor. These results for the localization of various spin fields coincide with the corresponding ones [11] in the Randall-Sundrum model [5] and many brane modes [24,25] except the spin-1 vector field. It is of interest that there is no localized vector field on the brane in the domain wall model,[3] while the vector field can be localized on the defect in the stringlike model. This phenomenon can be briefly explained as follows: In the Randall-Sundrum model, we can see that the

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[1]In a supersymmetric model, flat directions could appear so that the stability problem of moduli seems at first glance to be not so important as in a nonsupersymmetric model. But in this case, we need a fine-tuning of the parameters.

[2]In this terminology, topological defects with codimensions 3 and 4, respectively, would be called a monopolelike defect and an instantonlike defect.

[3]See Ref. [26] for an interesting possibility of electric charge nonconservation in a brane world where a higher-dimensional generalization of the Randall-Sundrum model is used in order to localize gauge fields on a brane.

overall coefficient in front of the gauge field action is divergent so that we do not have a normalizable zero mode of the bulk gauge field. On the other hand, in our stringlike model, we have an additional warped factor coming from part of the angular variable in the background metric in addition to the conventional warped factor. Combined with these two warped factors, the coefficient in front of the action becomes finite, so the zero mode of the bulk gauge field is normalizable and is consequently localized on the stringlike defect.

One aim of the present paper is to investigate this interesting property of bulk bosonic fields in the stringlike defect model in more detail. The case of a spin-2 graviton field has been already examined in Refs. [18,23], so we will concentrate on the study of spin-0 scalar and spin-1 vector fields. We will show that there are an infinite number of massless Kaluza-Klein (KK) modes which are degenerate with respect to the radial quantum number, but only one massless field among them is coupled to fermions on the stringlike defect. Moreover, the KK excitations of the gauge field have vanishing coupling to spin-1/2 fermions on the defect, so the gauge field can exist in the bulk without meeting any phenomenological constraints on the model, which should be contrasted with the Randall-Sundrum domain wall model where the strong coupling of the KK excitations of gauge fields to the brane fermion gave rise to a potential internal inconsistency within the theory [8,9].

This paper is organized as follows. In the next section, we review a stringlike defect solution with codimension 2. In Sec. III, the Kaluza-Klein decomposition of scalar fields is studied in a background obtained in Sec. II. Then, in Sec. IV, the procedure used in Sec. III is applied to the case of gauge fields. The final section is devoted to a discussion.

II. STRINGLIKE DEFECT

Let us start with a brief review of a stringlike defect solution to Einstein's equations with sources to fix our notation and conventions [22]. We consider Einstein's equations with a bulk cosmological constant Λ and an energy-momentum tensor T_{MN} in general D dimensions:

$$R_{MN}-\frac{1}{2}g_{MN}R=-\Lambda g_{MN}+\kappa_D^2 T_{MN}, \tag{1}$$

where κ_D denotes the D-dimensional gravitational constant. Throughout this article we follow the standard conventions and notation of the textbook of Misner, Thorne, and Wheeler [27].

Let us adopt the following metric *Ansatz*:

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + \tilde{g}_{ab}dx^a dx^b \\ &= e^{-A(r)}\hat{g}_{\mu\nu}dx^\mu dx^\nu + dr^2 + e^{-B(r)}d\Omega_{n-1}^2, \end{aligned} \tag{2}$$

where $M,N,\ldots$ denote D-dimensional space-time indices, $\mu,\nu,\ldots$ p-dimensional brane ones, and $a,b,\ldots$ n-dimensional extra spatial ones, so the equality $D=p+n$ holds. (We assume $p\geqslant 4$.) And $d\Omega_{n-1}^2$ stands for the metric on a unit $(n-1)$-sphere, which is concretely expressed in terms of the angular variables θ_i as

$$\begin{aligned} d\Omega_{n-1}^2 &= d\theta_2^2 + \sin^2\theta_2 d\theta_3^2 + \sin^2\theta_2 \sin^2\theta_3 d\theta_4^2 + \cdots \\ &\quad + \prod_{i=2}^{n-1}\sin^2\theta_i d\theta_n^2. \end{aligned} \tag{3}$$

Moreover, we shall take the *Ansatz* for the energy-momentum tensor respecting the spherical symmetry:

$$\begin{aligned} T^\mu_\nu &= \delta^\mu_\nu t_o(r), \\ T^r_r &= t_r(r), \\ T^{\theta_2}_{\theta_2} &= T^{\theta_3}_{\theta_3} = \cdots = T^{\theta_n}_{\theta_n} = t_\theta(r), \end{aligned} \tag{4}$$

where t_i $(i=o,r,\theta)$ are functions of only the radial coordinate r.

With these *Ansätze*, after a straightforward calculation, Einstein's equations (1) reduce to

$$e^A\hat{R} - \frac{p(n-1)}{2}A'B' - \frac{p(p-1)}{4}(A')^2 - \frac{(n-1)(n-2)}{4}(B')^2 + (n-1)(n-2)e^B - 2\Lambda + 2\kappa_D^2 t_r = 0, \tag{5}$$

$$e^A\hat{R} + (n-2)B'' - \frac{p(n-2)}{2}A'B' - \frac{(n-1)(n-2)}{4}(B')^2 + (n-2)(n-3)e^B + pA'' - \frac{p(p+1)}{4}(A')^2 - 2\Lambda + 2\kappa_D^2 t_\theta = 0, \tag{6}$$

$$\frac{p-2}{p}e^A\hat{R} + (p-1)\left(A'' - \frac{n-1}{2}A'B'\right) - \frac{p(p-1)}{4}(A')^2 + (n-1)\left[B'' - \frac{n}{4}(B')^2 + (n-2)e^B\right] - 2\Lambda + 2\kappa_D^2 t_o = 0, \tag{7}$$

where the prime denotes the differentiation with respect to r, and $\hat{R}$ is the scalar curvature associated with the brane metric $\hat{g}_{\mu\nu}$. Here we define the cosmological constant on the $(p-1)$-brane, Λ_p, by the equation

$$\hat{R}_{\mu\nu}-\frac{1}{2}\hat{g}_{\mu\nu}\hat{R}=-\Lambda_p\hat{g}_{\mu\nu}. \tag{8}$$

In addition, the conservation law for the energy-momentum tensor, $\nabla^M T_{MN}=0$, takes the form

$$t_r'=\frac{p}{2}A'(t_r-t_o)+\frac{n-1}{2}B'(t_r-t_\theta). \tag{9}$$

Our purpose is to find a stringlike defect solution, that is, $n=2$, with a warp factor $A(r)=cr$ (c is a positive constant) to the above equations. (The case of $n=1$ corresponds to a domain wall solution.) The necessity of this exponentially decreasing warp factor is to bind gravity to the p-brane. For generality, we consider a general space-time dimension D and a general brane dimension p with $D=p+2$, but the physical interest, of course, lies in the case of six space-time dimensions $(D=6)$ and a three-brane $(p=4)$. In the case of $n=2$, under the *Ansatz* $A(r)=cr$, Einstein's equations (5), (6), (7) are of the form

$$e^{cr}\hat{R}-\frac{p}{2}cB'-\frac{p(p-1)}{4}c^2-2\Lambda+2\kappa_D^2 t_r=0, \tag{10}$$

$$e^{cr}\hat{R}-\frac{p(p+1)}{4}c^2-2\Lambda+2\kappa_D^2 t_\theta=0, \tag{11}$$

$$\frac{p-2}{p}e^{cr}\hat{R}-\frac{p-1}{2}cB'-\frac{p(p-1)}{4}c^2+B''-\frac{1}{2}(B')^2-2\Lambda+2\kappa_D^2 t_o=0, \tag{12}$$

and the conservation law takes the form

$$t_r'=\frac{p}{2}c(t_r-t_o)+\frac{1}{2}B'(t_r-t_\theta). \tag{13}$$

From these equations, general solutions can be found as follows:

$$ds^2=e^{-cr}\hat{g}_{\mu\nu}dx^\mu dx^\nu+dr^2+e^{-B(r)}d\theta^2, \tag{14}$$

where

$$B(r)=cr+\frac{4}{pc}\kappa_D^2\int^r dr(t_r-t_\theta), \tag{15}$$

$$c^2=\frac{1}{p(p+1)}(-8\Lambda+8\kappa_D^2\alpha),$$

$$\hat{R}=\frac{2p}{p-2}\Lambda_p=-2\kappa_D^2\beta. \tag{16}$$

Here t_θ must take a definite form, which is given by

$$t_\theta=\beta e^{cr}+\alpha, \tag{17}$$

with α and β being some constants. Moreover, in order to guarantee the positivity of c^2, α should satisfy an inequality $-8\Lambda+8\kappa_D^2\alpha>0$.

Two types of special solution deserve more scrutiny. A specific solution is the one without sources $(t_i=0)$. Then we get a special solution which was found for a *local* string in Ref. [18] and for a *global* string in Ref. [15]:

$$ds^2=e^{-cr}\hat{g}_{\mu\nu}dx^\mu dx^\nu+dr^2+R_0^2e^{-cr}d\theta^2, \tag{18}$$

with R_0 being a length scale which we take to be of order unity. Here the positive constant c, the brane scalar curvature, and the brane cosmological constant are, respectively, given by

$$c^2=\frac{-8\Lambda}{p(p+1)},$$

$$\hat{R}=\frac{2p}{p-2}\Lambda_p=0. \tag{19}$$

In this case, as in the corresponding domain wall solution, the bulk geometry is the anti–de Sitter space, and the brane geometry is Ricci flat with vanishing cosmological constant. It has been recently found that this special solution corresponds to a *local* defect in the sense that the energy-momentum tensor is strictly vanishing outside the string core [18,23]

Another specific solution occurs when we have the spontaneous symmetry breakdown $t_r=-t_\theta$ [17]:

$$ds^2=e^{-cr}\hat{g}_{\mu\nu}dx^\mu dx^\nu+dr^2+R_0^2e^{-c_1r}d\theta^2, \tag{20}$$

where

$$c^2=\frac{1}{p(p+1)}(-8\Lambda+8\kappa_D^2 t_\theta)>0,$$

$$c_1=c-\frac{8}{pc}\kappa_D^2 t_\theta,$$

$$\hat{R}=\frac{2p}{p-2}\Lambda_p=0. \tag{21}$$

Notice that this solution is more general than the previous one (18) since this solution reduces to Eq. (18) when $t_\theta=0$. In Ref. [23], the solution (20) was called a *global* defect since there appears a hedgehog-type configuration outside the string core.

To close this section, let us comment on an interesting *global* defect recently found in a general dimension in Ref. [23]. To gain the *global* topological defect, the antisymmetric tensor field with rank $n-2$ is added to the Einstein-Hilbert action with a cosmological constant. Then the energy-momentum tensor associated with the $(n-2)$-form field in the bulk has the property

$$t_0=t_r=-t_\theta. \tag{22}$$

The *Ansatz* taken in Ref. [23] is

$$A(r)=cr,\quad B(r)=\text{const.} \tag{23}$$

With this *Ansatz* (23), it is easy to see that Einstein's equations (5), (6), (7) and the conservation law (9) require important equations

$$t_0=t_r=\text{const},\qquad t_\theta=\text{const}, \tag{24}$$

in addition to the other inessential equations for the present consideration. These conditions (24) are more general than Eq. (22), so if an energy-momentum tensor satisfies Eqs. (24), Einstein's equations with such an energy-momentum tensor would admit the *global* topological defect with background metric (23) as a solution in a general space-time dimension. Finally, note that this new *global* defect has the same property as the domain wall with respect to the localization of various bulk fields.

III. KALUZA-KLEIN DECOMPOSITION OF SCALAR FIELD

In a previous paper, it was shown that spin-0, -1, and -2 bosonic fields are localized on the p-brane defect with the exponentially decreasing warp factor, while spin-1/2 and =3/2 fermionic fields are not so in the stringlike defect [22]. Thus, it is natural to consider first the case of a bulk scalar field. The case of a bulk vector field will be examined in the next section. The spin-2 graviton was examined in detail in Ref. [18], so we skip this case in this paper. From now on, for clarity we shall limit our attention to a *local* stringlike solution (18) since the generalization to a *global* solution (20) is straightforward. Of course, we have implicitly assumed that various bulk fields considered below make little contribution to the bulk energy so that the solution (18) remains valid even in the presence of bulk fields.

Let us consider the action of a massless real scalar coupled to gravity,

$$S_\Phi=-\frac{1}{2}\int d^Dx\sqrt{-g}g^{MN}\partial_M\Phi\partial_N\Phi, \tag{25}$$

from which the equation of motion can be derived:

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\Phi)=0. \tag{26}$$

From now on we shall take $\hat{g}_{\mu\nu}=\eta_{\mu\nu}$ and define $P(r)=e^{-cr}$. In the background metric (18), the equation of motion (26) reads

$$P^{-1}\eta^{\mu\nu}\partial_\mu\partial_\nu\Phi+P^{-(p+1)/2}\partial_r(P^{(p+1)/2}\partial_r\Phi)+\frac{1}{R_0^2}P^{-1}\partial_\theta^2\Phi=0. \tag{27}$$

Let the KK expansion of Φ be given by

$$\Phi(x^M)=\sum_{n,l=0}^{\infty}\phi^{(n,l)}(x^\mu)\frac{\chi_n(r)}{\sqrt{R_0}}Y_l(\theta). \tag{28}$$

Here $Y_l(\theta)$ are in general the eigenfunction of the scalar Laplacian Δ on a unit $(n-1)$-sphere with the eigenvalues $l(l+n-2)$. Now we are taking account of a stringy defect with codimension 2; i.e., n is chosen to 2, so we have an equation

$$\Delta Y_l(\theta)=l^2Y_l(\theta), \tag{29}$$

with $l=0,1,2,\ldots$. And $Y_l(\theta)$ satisfy the following orthonormality condition:

$$\int_0^{2\pi}d\theta Y_l(\theta)Y_{l'}(\theta)=\delta_{ll'}. \tag{30}$$

Using the KK expansion (28), the equation of motion (27) reduces to the well-known Klein-Gordon equation with the KK masses m_n,

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu-m_n^2)\phi^{(n,l)}=0, \tag{31}$$

where we have required χ to satisfy the following differential equation:

$$-\left(P^{-(p-1)/2}\partial_rP^{(p+1)/2}\partial_r-\frac{l^2}{R_0^2}\right)\chi_n=m_n^2\chi_n. \tag{32}$$

Actually, it is easily shown that by means of Eqs. (28), (29), (30), and (32) the starting action (25) can be written as

$$S_\Phi=-\frac{1}{2}\sum_{n,l=0}^{\infty}\int d^px[\eta^{\mu\nu}\partial_\mu\phi^{(n,l)}\partial_\nu\phi^{(n,l)}+m_n^2\phi^{(n,l)}\phi^{(n,l)}], \tag{33}$$

where we have also used the orthonormality condition

$$\int_0^\infty dr P^{(p-1)/2}\chi_n\chi_{n'}=\delta_{nn'}. \tag{34}$$

To analyze the scalar KK mass spectrum, it is necessary to solve Eq. (32) explicitly. Defining $M_n^2=m_n^2-l^2/R_0^2$, $z_n=(2/c)\,M_nP^{-1/2}$ and $h_n=P^{(p+1)/4}\chi_n$, Eq. (32) can be written in the form

$$\left[\frac{d^2}{dz_n^2}+\frac{1}{z_n}\frac{d}{dz_n}+\left\{1-\frac{1}{z_n^2}\left(\frac{p+1}{2}\right)^2\right\}\right]h_n=0, \tag{35}$$

which is nothing but the Bessel equation of order $(p+1)/2$. Thus, the solutions are of the form

$$\chi_n=\frac{1}{N_n}P^{-(p+1)/4}[J_{(p+1)/2}(z_n)+\alpha_nY_{(p+1)/2}(z_n)], \tag{36}$$

where N_n are the wave function normalization constants and α_n are constant coefficients. The differential operator in Eq. (32) is self-adjoint provided that one imposes the boundary conditions [18]

$$\chi_n'(0)=\chi_n'(\infty)=0. \tag{37}$$

These boundary conditions lead to the relations

$$\alpha_n=-\frac{J_{(p-1)2}(z_n(0))}{Y_{(p-1)/2}(z_n(0))}=-\frac{J_{(p-1)/2}(z_n(\overline{r}))}{Y_{(p-1)/2}(z_n(\overline{r}))}, \tag{38}$$

where $\overline{r}$ indicates the infrared cutoff, which is taken to be an infinity at the end of calculations. Incidentally, in deriving Eq. (38) we have used the formula holding in the Bessel functions:

$$Z_\nu'(z)=Z_{\nu-1}(z)-\frac{\nu}{z}Z_\nu(z), \tag{39}$$

with Z being J or Y. Now in the limit $M_n\ll c$, the KK masses can be derived from the equation [9]

$$J_{(p-1)/2}(z_n(\overline{r}))=0, \tag{40}$$

which gives us the approximate mass formula

$$M_n=\frac{c}{2}\left(n+\frac{p}{4}-\frac{1}{2}\right)\pi e^{-c\overline{r}/2}. \tag{41}$$

Moreover, the normalization constant N_n takes the approximate form in the limit $M_n\ll c$,

$$N_n=\sqrt{c}\,\frac{z_n(\overline{r})}{2M_n}J_{(p+1)/2}(z_n(\overline{r})). \tag{42}$$

Note that in the limit $\overline{r}\to\infty$, M_n approach zero as in the graviton [18], which is a characteristic feature of noncompact extra dimensions. The KK masses of a scalar field are given by not M_n but m_n, so it turns out that they approach l^2/R_0^2. Accordingly, only the s-wave $(l=0)$ mode becomes massless on the stringlike defect while the other modes are massive. Here it is worth noticing that the massless s-wave mode is degenerate with respect to the radial quantum number n since the KK masses depend on only l in the limit $\overline{r}\to\infty$. Thus, we are in danger of the existence of an infinite number of massless modes on the defect, which seems to be against the phenomenology. Luckily enough, however, as will be shown below, only a unique massless mode with $n=0$ couples to fermions on the defect since the coupling constant between the remaining massless modes with $n\geqslant 1$ and the defect fermion vanishes in the infinite volume limit. It would be then natural to identify this massless zero mode with $n=0$ as the Higgs field in our world from the phenomenological viewpoint.

As is shown in Ref. [22], the spin-1/2 fermion is localized on a defect with an exponentially rising warp factor, so it is necessary to invoke additional interactions except gravity in a model in order to localize the spin-1/2 fermion on our defect, which has the exponentially decreasing warp factor. In this paper, we simply consider fermions on the stringlike defect.

To see that only the massless zero mode with $n=0$ couples to fermions on the defect, it is useful to examine the Yukawa coupling whose interaction term is given by

$$S_{\overline{\Psi}\Psi\Phi}=-g_\Phi\int d^Dx\sqrt{-g}\,\overline{\Psi}\Psi\Phi\,\delta(r). \tag{43}$$

The integration over the angular variable and the KK expansion (28) yield

$$S_{\overline{\Psi}\Psi\Phi}=-g_\Phi\sqrt{R_0}\int d^px\overline{\Psi}\Psi\sum_{n=0}^{\infty}\phi^{(n,0)}(x)\chi_n(0). \tag{44}$$

The wave function for the zero mode is a constant and from the orthonormality condition (34) we have the zero mode

$$\chi_0=\sqrt{\frac{c(p-1)}{2}}. \tag{45}$$

For the excited KK modes $\chi_n(0)$ with $n\geqslant 1$, it is easy to evaluate $\chi_n(0)$ in the limit $M_n\ll c$:

$$\chi_n(0)=\frac{1}{N_n}J_{(p+1)/2}\left(\frac{2}{c}M_n\right)=\sqrt{c}P^{1/4}(\overline{r}), \tag{46}$$

where Eq. (42) was used. Moreover, defining the effective p-dimensional coupling constant as $\tilde{g}_\Phi=g_\Phi\sqrt{[c(p-1)/2]\ R_0}$, the Yukawa interaction can be expressed as

$$S_{\overline{\Psi}\Psi\Phi}=-\tilde{g}_\Phi\int d^px\overline{\Psi}\Psi\left[\phi^{(0,0)}(x)+\sqrt{\frac{2}{p-1}}P^{1/4}(\overline{r})\sum_{n=1}^{\infty}\phi^{(n,0)}(x)\right]. \tag{47}$$

From this equation, it is obvious that the effective coupling of the excited KK modes with $n\geqslant 1$ to the defect fermion vanishes in the limit $\overline{r}\to\infty$ owing to the presence of $P^{1/4}(\overline{r})$ in front of the second term. On the other hand, the massless zero mode with $n=0$ has a coupling constant of order 1. Hence, only the massless zero mode resides in the stringlike defect.

IV. KALUZA-KLEIN DECOMPOSITION OF VECTOR FIELD

Next we turn our attention to the case of vector field. It was shown in the Randall-Sundrum model in AdS_5 space that the spin-1 vector field is not localized either on a brane with positive tension or on a brane with negative tension, so the Dvali-Shifman mechanism [28] must be invoked for the vector field localization [11,9]. On the other hand, we have shown that the spin-1 vector field *is* localized on a stringlike defect like spin-0 scalar and spin-2 graviton fields [22]. So we do not need to introduce additional mechanisms for the vector field localization in the case at hand. The localization of the vector field on the defect therefore allows us to think of the bulk vector field.

Let us start with the action of the $U(1)$ vector field:

$$S_A=-\frac{1}{4}\int d^D x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \tag{48}$$

where $F_{MN}=\partial_M A_N-\partial_N A_M$ as usual. (The extension to the case of non-Abelian gauge fields is straightforward.) From this action the equations of motion are given by

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS})=0. \tag{49}$$

With the background metric (18) and the gauge conditions $\partial_\mu A^\mu=A_\theta=0$, these equations become

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu+P^{(3-p)/2}\partial_r P^{(p-1)/2}\partial_r+\frac{1}{R_0^2}\partial_\theta^2\right)A_\lambda - P^{(3-p)/2}\partial_r P^{(p-1)/2}\partial_\lambda A_r=0, \tag{50}$$

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu+\frac{1}{R_0^2}\partial_\theta^2\right)A_r=0, \tag{51}$$

$$\partial_r(P^{(p-1)/2}\partial_\theta A_r)=0. \tag{52}$$

Let us take the following forms of the KK decomposition for later convenience:

$$A_\mu(x^M)=\sum_{n,l=0}^{\infty}A_\mu^{(n,l)}(x^\mu)\frac{f_n(r)}{\sqrt{R_0}}Y_l(\theta),$$

$$A_r(x^M)=\sum_{l=0}^{\infty}A_r^{(l)}(x^\mu)\frac{g(r)}{\sqrt{R_0}}Y_l(\theta). \tag{53}$$

Then, from Eq. (51) we have

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu-\frac{l^2}{R_0^2}\right)A_r^{(l)}(x^\mu)=0. \tag{54}$$

In addition, Eq. (52) leads to a general solution for $g(r)$:

$$g(r)=\alpha P^{-(p-1)/2}, \tag{55}$$

with α being an integration constant. Finally, using Eq. (55), Eq. (50) reduces to the form

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu-m_n^2)A_\lambda^{(n,l)}(x^\mu)=0. \tag{56}$$

Here we have required $f_n(r)$ to satisfy the differential equation

$$-\left(P^{-(p-3)/2}\partial_r P^{(p-1)/2}\partial_r-\frac{l^2}{R_0^2}\right)f_n(r)=m_n^2 f_n(r). \tag{57}$$

As in the case of a scalar field, let us substitute the KK expansion (53) and the solution (55) into the starting action (48), whose result is given by

$$S_A=\int d^p x\sum_{n,l=0}^{\infty}\left[-\frac{1}{4}\eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}^{(n,l)}F_{\nu\rho}^{(n,l)} -\frac{1}{2}m_n^2\eta^{\mu\nu}A_\mu^{(n,l)}A_\nu^{(n,l)}\right]-\frac{\alpha^2}{c(p-1)} \times[P(\overline{r})^{-(p-1)/2}-1]\int d^p x \times\sum_{l=0}^{\infty}\left[\eta^{\mu\nu}\partial_\mu A_r^{(l)}\partial_\nu A_r^{(l)}+\frac{l^2}{R_0^2}A_r^{(l)}A_r^{(l)}\right], \tag{58}$$

where we have used Eq. (30) and the orthonormality condition for $f_n(r)$:

$$\int_0^\infty dr P^{(p-3)/2}f_n f_{n'}=\delta_{nn'}. \tag{59}$$

Note that the coefficient in front of the action of the scalar field A_r becomes divergent in the limit $\overline{r}\to\infty$, but this divergence can be absorbed in the redefinition of the field $A_r^{(l)}$. That is, by performing the field redefinition

$$A_r^{(l)}\to\alpha\sqrt{\frac{2}{c(p-1)}}\sqrt{P(\overline{r})^{-(p-1)/2}-1}A_r^{(l)},$$

we arrive at the expression

$$S_A=\int d^p x\sum_{n,l=0}^{\infty}\left[-\frac{1}{4}\eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}^{(n,l)}F_{\nu\rho}^{(n,l)} -\frac{1}{2}m_n^2\eta^{\mu\nu}A_\mu^{(n,l)}A_\nu^{(n,l)}\right]-\frac{1}{2}\int d^p x \times\sum_{l=0}^{\infty}\left[\eta^{\mu\nu}\partial_\mu A_r^{(l)}\partial_\nu A_r^{(l)}+\frac{l^2}{R_0^2}A_r^{(l)}A_r^{(l)}\right]. \tag{60}$$

Here notice that the second integral describes that the ''gauge scalar'' has the same structure as the action of the scalar field (33). Thus, following an argument similar to the case of a scalar field, it is straightforward to show that only the massless zero mode of the ''gauge scalar'' couples to the fermion on the defect. Therefore, in what follows, we shall consider the p-dimensional gauge field A_μ.

To examine the KK spectrum, we can follow a path of argument similar to that of a scalar field in Sec. III. This time, by defining $h_n = P^{(p-1)/4} f_n$, Eq. (57) can be written as

$$\left[\frac{d^2}{dz_n^2}+\frac{1}{z_n}\frac{d}{dz_n}+\left\{1-\frac{1}{z_n^2}\left(\frac{p-1}{2}\right)^2\right\}\right]h_n=0, \qquad (61)$$

whose solution is also expressed in terms of the Bessel functions of order $(p-1)/2$:

$$f_n(z_n)=\frac{1}{N_n}P^{-(p-1)/4}[J_{(p-1)/2}(z_n)+\alpha_n Y_{(p-1)/2}(z_n)], \qquad (62)$$

where N_n are new wave function normalization constants and α_n are new constant coefficients. The same boundary conditions (37) for $f_n(r)$ lead to the relations

$$\alpha_n=-\frac{J_{(p-3)/2}(z_n(0))}{Y_{(p-3)/2}(z_n(0))}=-\frac{J_{(p-3)/2}(z_n(\bar{r}))}{Y_{(p-3)/2}(z_n(\bar{r}))}. \qquad (63)$$

In the limit $M_n \ll c$, the KK masses can be derived from the equation

$$J_{(p-3)/2}(z_n(\bar{r}))=0, \qquad (64)$$

which gives us the approximate mass formula

$$M_n=\frac{c}{2}\left(n+\frac{p}{4}-1\right)\pi e^{-c\bar{r}/2}, \qquad (65)$$

And the normalization constant N_n takes the approximate form

$$N_n=\sqrt{c}\,\frac{z_n(\bar{r})}{2M_n}J_{(p-1)/2}(z_n(\bar{r})). \qquad (66)$$

Note that in the limit $\bar{r}\to\infty$, the KK masses of vector field are given by l^2/R_0^2 like the scalar case. Hence, as expected, only the s wave ($l=0$) becomes massless on the p-brane defect while the other modes are massive. This time, compared to the scalar case, it is more important to show that only one massless mode with $n=0$ resides in the defect since such a massless zero mode would be regarded as a unique ''photon'' on the defect.

We are now ready to consider the coupling of the gauge KK modes to spin-1/2 fermions on the p-brane defect. The fermion kinetic and gauge interaction terms are given by

$$S_\Psi=\int d^D x\sqrt{-g}\,\bar{\Psi}i\Gamma^M(\partial_\mu+ig_A A_\mu)\Psi\,\delta^\mu_M\delta(r), \qquad (67)$$

where the curved gamma matrices Γ^μ and the flat gamma ones γ^μ are related through the relations $\Gamma^\mu=P^{-1/2}\gamma^\mu$. Upon integrating over θ and using the KK expansion (53), we obtain, for the gauge-fermion interaction term,

$$S_{\bar{\Psi}\Psi A}=-g_A\sqrt{R_0}\int d^p x\,\bar{\Psi}\gamma^\mu\sum_{n=0}^{\infty}A_\mu^{(n,0)}(x)f_n(0)\Psi. \qquad (68)$$

The wave function for the zero mode is again a constant and the orthonormality condition (59) gives us the zero mode

$$f_0=\sqrt{\frac{c(p-3)}{2}}. \qquad (69)$$

Recall that this zero mode is localized on the stringlike defect and is identified with the usual ''photon'' of the p-dimensional Minkowski space-time [22] while it is not localized on the domain wall [9,11]. For the excited KK modes $f_n(0)$ with $n\geqslant 1$, it is easy to evaluate $f_n(0)$ in the limit $M_n \ll c$:

$$f_n(0)=\frac{1}{N_n}J_{(p-1)/2}\left(\frac{2}{c}M_n\right)=\sqrt{c}P^{1/4}(\bar{r}), \qquad (70)$$

where Eq. (66) was used. Then, defining the effective p-dimensional $U(1)$ coupling constant as $\tilde{g}_A = g_A[\sqrt{c(p-3)/2}]R_0$, the interaction term reads

$$S_\Psi^{int}=-\tilde{g}_A\int d^p x\,\bar{\Psi}\gamma^\mu\left[A_\mu^{(0,0)}(x)+\sqrt{\frac{2}{p-3}}P^{1/4}(\bar{r})\sum_{n=1}^{\infty}A_\mu^{(n,0)}(x)\right]\Psi. \qquad (71)$$

From this equation, it is obvious that the coupling of the excited KK modes to the defect fermion vanishes in the limit $\bar{r}\to\infty$ owing to the presence of $P^{1/4}(\bar{r})$ as in the scalar field. On the other hand, the massless zero mode has a coupling constant of order 1 as desired. Thus, this model is consistent with gauge fields existing in the bulk, which should be contrasted with the Randall-Sundrum model [8,9]. Of course, further studies are necessary to assure the consistency of the model at hand at the quantum level.

V. DISCUSSIONS

In this paper we have explored the possibility of placing a spin-0 scalar field and spin-1 vector gauge field in the bulk in the stringlike defect model in detail. We have derived the scalar and the gauge field KK spectra from examination of the action of the theory and also analyzing the equations of motion.

We then computed the scalar-fermion and the gauge-fermion interactions on the stringlike defect and found that the excited KK states with respect to the radial quantum number do not couple to fermion on the defect in the infinite cutoff limit, whereas the massless zero modes, which are nothing but the ''Higgs'' particle and the usual ''photon'' of the Minkowski space-time in the cases of scalar and gauge bosons, respectively, couple to fermions with order unity. Since it has been already shown that the spin-2 graviton is localized on the defect [22] and yields the desired Newton's law with tiny correction terms on the defect [18], the model which we consider equips us with desirable physical properties. Of course, to make the model at hand more realistic we need additional interactions except gravity for localizing fermions on the defect, but we wish to insist that a supergravity model corresponding to the present model would resolve this problem in a natural way even if we could localize fermionic fields by introducing additional interaction terms between bosons and fermions by hand.

Let us restrict the following argument to the case of $p = 4$ and $D = 6$. In this case, it is of interest to imagine that 10D superstring theory might be compactified on a Calabi-Yau twofold manifold, i.e., K3 by the conventional Kaluza-Klein mechanism, yielding 6D theory, and then the 6D theory is compactified to our four-dimensional space-time according to the alternative compactification scenario discussed in this paper. Here it is worthwhile to mention that the 6D local field theory is a very interesting field theory. (Correspondingly, 6D supergravity theory possesses a richer structure than 5D supergravity theory in many respects.) For instance, the 6D local field theory is a free theory with a trivial cubic scalar self-interaction with unrenormalizable Einstein-Hilbert and Yang-Mills actions, so it is expected that ''little'' superstring theory may play an essential role. Furthermore, our model in six space-time dimensions has a physical setting where our world is a three-brane embedded in 6D space-time with nonfactorizable warped geometry. Interestingly enough, the metric of two internal dimensions is conformally flat, so the powerful (Euclidean) conformal field theory technique can be naturally applied to our model.

Finally let us comment on a supersymmetric realization of the present model. A lot of attention has been recently devoted to the construction of a supersymmetric Randall-Sundrum model [29–35] and advocated some no-go theorems. Then it is of interest to ask whether there are supersymmetric relations between our 6D model and the 5D Randall-Sundrum model. Through a simple KK dimensional reduction, it seems that the background metric in our model reduces to the one in the Randall-Sundrum model. At the same time, $N = 2$, 6D supergravity would reduce to $N = 2$, 5D supergravity. Thus, if we cannot construct a supersymmetric version of the Randall-Sundrum model, it might be also difficult to construct a supersymmetric model of our 6D theory. But there recently appeared an interesting construction of a supersymmetric Randall-Sundrum model [35]. The corresponding construction of our 6D model is now being actively investigated, so we hope to report on this construction in the near future.

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